Use of Graph Theory to support the evaluation of the resilience of urban water infrastructure

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1. Surrogate measures for water network resilience
2. Working with large-scale networks
3. Conclusions
Although there are no formal definitions for quantifying the resilience of water distribution networks (WDNs), a common method is to formulate the hydraulic resilience as a measure of the ability of a network to maintain supply under failure conditions. Todini (2000) proposed a resilience index based on the steady state flow analysis of WDNs and dissipated energy; consequently, the resilience of a water network was defined using a measure of the available surplus energy.

Large / complex networks requires to take surrogate measures of the resilience.

After a number of surrogate measures of the Todini’s resilience index that present difficulties to work in large size or not well calibrated networks, an important contribution to the application of complex networks theory to water networks is Yazdani and Jeffrey (2012).
**Graph-theoretic framework**

- Novel graph-theoretic framework
  - Hydraulic head losses
  - Resistance routes / capacity of supply

- Avoids the requirement of iterative hydraulic simulations
  - Topological significance of pipes and nodes
  - Steady state hydraulic simulation

- Automatic classification on system criticality and resilience
  - Resilience for critical customers
New indices: essentials

- **Edge betweenness centrality**: measures the degree to which a link is between pairs of vertices by shortest paths connecting them, that is, the number of times that the link appears in the outcome of all the possible shortest paths.
  - Our proposal adds the flow rate passing through the pipes connecting all the consumption nodes (vertices of the WDN) and compute the importance of each different pipe quantifying the flow that pass through it together with the ease of supply of every path.

- **Closeness centrality**: measures the average distance between the network vertices. The information that this measure provides is on the density of nodes that exists in a network (or in different areas of a network), together with an idea of how well each node is connected with the network in terms of geodesic distance.
  - Our proposal enhances this classical version of the closeness measure providing hydraulic meaning to that distance between nodes in terms of head losses.
Adapting graph-theoretic indices to water networks resilience (i)

- Detecting customers poorly connected (sensible to any failure)
  - Random-walk closeness
  - Hydraulic head losses

W-Fc is defined by

\[
c_{WC} = \frac{n_C}{\sum_{t \neq v} \lambda(t, v)}
\]  

(1)

\[
\lambda(tv) = \min_{l(t,v)} \left\{ \sum_{j=1}^{m} \text{sign}(q_j)r_jq_j^2 \right\}
\]

(2)

where \( l(t, v) \) represents the set of all the paths connecting the nodes \( t \) and \( v \).
Adapting graph-theoretic indices to water networks resilience (ii)

- Availability and capacity of routes to source nodes
  - K shortest paths
  - Hydraulic pipe capacities

KSP for a node is

\[
I_{GT}(v) = \sum_{s=1}^{S} \left( \frac{1}{K} \sum_{k=1}^{K} \frac{1}{g(k, s)} \right)
\]

(3)

with energy loss is given as:

\[
g(k) = \sum_{m=1}^{M} \frac{L_m}{D_m^\alpha}
\]

(4)
C-Town network

- 333 nodes
- 429 pipes and 4 valves
- 5 pump stations, 7 elevated tanks
- 1 reservoir

Comparison of water-flow closeness (W-Fc) and connectivity to water sources ($I_{GT}$) for all the C-Town network nodes
Water utility network

- 2,374 nodes
- 1,945 pipes and 489 valves
- 2 reservoirs
- 2 sectors

Comparison of water-flow closeness (W-Fc) and connectivity to water sources ($I_{GT}$) for all the network nodes
Aggregation of sectors/DMAs

Abstraction of the DMA features into a weighted graph:

- Layer \( j + 1 \): edge weights proportional to the number of links (pipes) between DMAs
- Layer \( j + 1 \): inheritance process by average measures in (super)nodes
- Layer \( j \): kernel spectral clustering to aggregate the more similar DMAs regarding hydraulic and graph characteristics into the same group
Large utility networks (i)

- Acquisition of networks topology data (GIS and hydraulic models)
- Visualization analytics to represent topological connectivity of large scale networks

- 4820 nodes
- 5234 pipes (and valves)
- 1 reservoir
- 3 DMAs

- 106,115 nodes
- 111,096 pipes
- 1,900 boundary valves
- 4 constant head sources
- 28 reservoirs
- 102 pumps
- 228 DMAs
Large utility networks (ii)

- Validation, assessment and multi-view visualization
1 Surrogate measures for water network resilience

2 Working with large-scale networks

3 Conclusions
Conclusions

The introduced measures take further typical graph theory indices that are based on geodesic shortest paths to make them more realistic in approximating true network resilience

- Water-Flow closeness complements graph centrality analyses suggesting areas of low resilience nodes
  - efficient and reliable for assessing water network resilience given that it just needs to solve a single steady state hydraulic equations

- Weighted KSP from consumption nodes to water sources have been used to estimate the abundance and capacity of supply routes of nodes to sources
  - the use of $K$ different routes for every node provides statistical robustness to our analysis; besides, water is mainly distributed by alternative pipelines to those just belonging to the shortest path
Conclusions

- The novel graph-theoretic framework is complimented by a novel multiscale decomposition method that converts the original WDN layout to DMA-based graphs.
- The developed method allows us to work with a WDN at two different levels of abstraction: single demand node and DMA.
- This enhances the results obtained for assessing the resilience of a WDN, especially in the case of large scale networks.


